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Computational Modal Analysis of the LASTA Aircraft

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This paper presents the results of the Computational Modal Analysis of a complete finite element model of the LASTA aircraft. The results of calculation, normal frequencies and normal mode shapes, shown here, will be used for the further aeroelastic analysis to confirm that this aircraft is free from flutter in the range of the designed speed. A short theoretical review of the Computational Modal Analysis and Lanczos Method is presented in this paper.

Key words: **aircraft, propeller aircraft, trainer aircraft, eigen oscillations, modal analysis, numerical analysis, Finite Element Method.**

Introduction

DURING designing, manufacturing and examination of an aircraft prototype it is required to confirm that the aircraft is free from flutter in the range of the designed speed. The flutter analysis and the flight test are used to confirm it.

This paper presents the results of the Computational Modal Analysis, i.e. the computation of the natural frequencies and the natural mode shapes of the LASTA aircraft by using the Finite Element Method. First, a finite element model of the LASTA aircraft is done. Then, the natural frequencies and the natural mode shapes are estimated using the Computational Modal Analysis. All this is done using the I-DEAS software. The results of the Computational Modal Analysis will be used for the Flutter Analysis, i.e. the calculation of unsteady generalized aerodynamic forces and critical flatter speeds of the LASTA aircraft.

Finite Element Method and Dynamic Analysis

Finite element modeling is used in the design cycle of new mechanical parts or systems because it offers a number of advantages [1], [2]. Using the Finite Element Method (FEM), a mathematical model of the structure can be built much more before the first prototype structure is even built. In addition, it can be determined how the mass or stiffness changes of the structure affect its static and dynamic characteristics and this is its great advantage.

All structures, regardless of their simplicity, have an infinitive number of degrees of freedom (DOF). With generalization of the matrix methods and with the advantages of modern computers, engineers started using the Finite Element Method in the static and dynamic analysis of large structures. One of the main objectives of selecting an FE model is to reduce the infinitive degree of freedom of systems with complex geometry and material properties, to a model with a limited DOF, but which will capture the significant physical behavior of the system. The Finite Element Method is one of the most powerful popular mathematics modeling techniques. In brief, the basis of the FEM is the representation of a body or a structure by an assemblage of subdivisions called finite elements. These elements are interconnected at joints, which are called nodes or nodal points. Simple functions are chosen to approximate the distribution or variation of the actual displacements over each finite element. Such assumed functions are called displacement functions. The final solutions yield to displacements at nodes. The attraction of the FEM is that it is commercially available in the form of a user-oriented computer program.

In practice, the finite element model of any structure approximates a real structure using finite elements. If the behavior of a structure is too complex, the recipe is to divide it into substructures that are more manageable [3]. If these substructures are still too complex, the subdivision process is continued. The first step in the Finite Element Method is to subdivide a structure, or an assembly, into many smaller elements such as plates, beams, shafts, etc., or substructures. The accuracy of the model depends on a degree of the users' skill in choosing the discretization. The most important is to choose an appropriate element type and size for the parts or substructures, correctly connect all parts and choose appropriate boundary conditions. Then the overall equations of motion of the structure are made from the equations describing the motions of each of individual parts, or substructures, plus all the boundary conditions at the connection points between parts or substructures.

The Finite Element Method is typically used for the following analyses:

- Static analysis (static strength to observe the high stress and strain areas under static loads),
- Dynamic analysis (modal analysis, dynamic responses to external dynamic loads, dynamic strength and fatigue).

One of the most important fields of using the Finite Element Method is in the dynamic analysis [4]. By using the finite element model, it is possible to simulate the response of the real world to an external stimulus. External forces might be of short duration and high amplitude, i.e. impulse forces, which cause immediate structure crash.

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Furthermore, they may be of long duration and cyclic in nature and could cause fatigue damage of the structure during a long period. In that way, it is possible to predict structure damage, its crash or fatigue life even before the first prototype is constructed.

In the static analysis, the element size is smaller and a total number of elements are larger. In the dynamic analysis, the element size is bigger because the calculation method is more complex and then the total number of elements is smaller. Time-varying characteristics make the dynamic analysis more complicated and more realistic than the static analysis [5]. Two basic aspects of the dynamic analysis, which differentiate it from the static analysis, are:

- Dynamic loads are applied as a function of time.
- Time-varying external dynamic load induces a time-varying response (displacements, velocities, accelerations, forces, and stresses).

The primary task for the dynamic analyst is to determine the type of analysis to be performed. The choice of the appropriate mathematical approach depends on the nature of the dynamic analysis in many cases. The extent of the information required from a dynamic analysis also dictates the necessary solution approach and steps.

Two basic classifications of the dynamic analysis are:

- free vibrations and
- forced vibrations.

The free vibration analysis is used to determine the basic dynamic characteristics of the system when the external forces are zero, i.e., there is no applied load. Free vibrations can be damped and undamped. If damping is neglected, than it is an undamped free vibration.

Natural Frequency and Natural Mode Shape Computational Modal Analysis

The determination of natural (normal) frequencies and natural (normal) mode shapes of the structure with damping neglected is the first step in the dynamic analysis. According to natural frequencies and natural mode shapes that characterize the basic dynamic behavior of the structure, we have the indication of how the structure will respond to dynamic loading. The natural frequencies of a structure are the frequencies at which the structure naturally tends to vibrate if subjected to a disturbance. An appropriate deformed shape of the structure at a specific natural frequency of vibration is its normal mode shape, or the normal mode of vibration. Natural frequencies and natural mode shapes of any structure are estimated owing to the Computational Modal Analysis using finite element model of that structure [6]. They depend on the structural properties and boundary conditions.

Computation of natural frequencies and natural mode shapes of a structure could be of great significance. They tell us at what frequencies the structure can be excited into resonant motion. In many cases, this information is sufficient for modifying the structural design in order to reduce noise and vibration. In addition, a dynamic interaction between a component and its supporting structure is very important because the component could cause structural damage or failure if its operating frequency is close to one of the natural frequencies of the structure. A transient response, frequency response, response spectrum and modal frequency analysis can be based on the results of the natural frequency and natural mode shapes. Another important field of use is in the experiment when the natural frequencies and natural mode shapes can be used to guide the experiment, for example to help the researchers to indicate at which frequencies to vibrate the structure and to indicate the best location for the accelerometers. In addition, design changes can also be evaluated by using natural frequencies and natural mode shapes. Finally, the knowledge of the natural frequencies and natural mode shapes for a particular structure is important for all types of dynamic analyses.

The dynamic response verification process [7] of the project is shown in Figure 1. The prototype can be virtual (finite element model) or real (physical prototype). Analytical, computational results (natural frequencies and natural mode shapes) are obtained based on the virtual prototype using the Computational Modal Analysis and the experimental results based on the real prototype using Experimental Modal Analysis, Vibration Tests, or Flight Tests. The verification of the finite element model and the results of the Computational Modal Analysis can be done after the experiments when it is possible to compare the modal parameters (natural frequencies and natural mode shapes) obtained computationally and experimentally. The correction of the finite element model and then natural frequencies and natural mode shapes can be done according to the experimental results, Fig.1. A new, calibrated virtual prototype may be further used for some other kinds of dynamic phenomena or for the dynamic analysis and design modification.

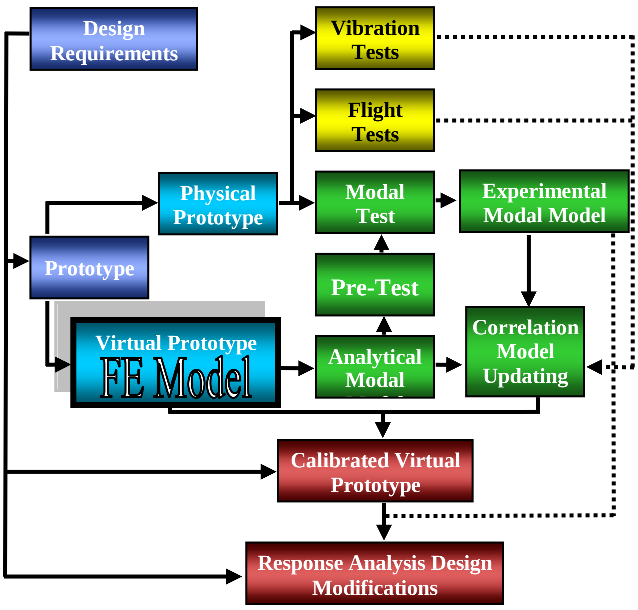

Figure 1. Dynamic response verification process

Theoretical review equation of motion for undamped free vibration

Natural frequencies and natural mode shapes are obtained as the results of the eigenvalue problem [8], i.e. equation of motion for undamped free vibrations.

The general equation of motion in the matrix form is:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{P\} \tag{1}$$

where:

$[M]$ - mass matrix

$[C]$ - damping matrix

$[K]$ - stiffness matrix

$\{P\}$ - force vector

$\{u\}$ - displacement vector

Disregarding the dissipative forces $[C]\{\dot{u}\}$ and when the external (excitation) forces are zero, $\{P\}$=0, the equation of motion (1) reduces to equation of motion for undamped free vibrations and its matrix form is:

$$[M]\{\ddot{u}\}+[K]\{u\}=0 \tag{2}$$

Dissipative forces are neglected because there is no possibility to estimate the damping matrix in this phase of the project. Damping effects can only be included approximately and that is way damping effects are neglected altogether in many analysis. Dumping is obtained experimentally after the Experimental Modal Analysis when natural frequencies and natural mode shapes are obtained as well.

Using the Modal Decomposition Method and the condition that it is a harmonic vibration $\{\ddot{u}\}=-\omega^2\{u\}$, eq.(2) reduces to the eigenvalue problem:

$$\left([K]-\omega^2[M]\right)\{\phi\}=0 \tag{3}$$

where:

$\{\phi\}$ - eigenvector, or natural mode shape

ω - circular natural frequency.

Eq. (3) is called eigenproblem and can be interpreted as a set of homogeneous algebraic equations. It represents base for the eigenvalue problem calculation. The first necessary step in the dynamic analysis of the structure is to solve the characteristic equation.

By pre-multiplying eigenequation (3) by the matrix $[M]^{-1}$, or by the matrix $[K]^{-1}$, it can be written as a standard eigenvalue problem:

$$[A]\{\phi\}=\lambda\{\phi\} \tag{4}$$

where:

$[A]=[M]^{-1}[K]$ - square dynamic matrix

$\lambda=\omega^2$ - eigenvalue

The matrix $[M]^{-1}$ must be non-singular. A consistent mass matrix always satisfies this condition. A stiffness matrix will satisfy this condition if the rigid body motion degrees of freedom are suspended. If the rigid body motion degrees of freedom exist, the Shift Frequency Method [9] is applied and it transforms eq. (3) to:

$$\left([K']-\left(\omega^2\pm S^2\right)[M]\right)\{\phi\}=0 \tag{5}$$

where:

$$[K']=[K]\pm S^2[M] \tag{6}$$

Eq. (3), eigenequation, has two possible solutions:

1. If $\det\left([K]-\omega^2[M]\right)\neq 0$, the only possible solution is $\{\phi\}$=0. This is a trivial solution, which does not give any useful information from a physical point of view, except that there is no motion.
2. If $\det\left([K]-\omega^2[M]\right)=0$, the solution is non-trivial $\{\phi\}\neq 0$.

The eigenvalue problem is then reduced to the following equation:

$$\det\left([K]-\omega^2[M]\right)=0 \tag{7}$$

and for $\omega^2=\lambda$ is:

$$\det\left([K]-\lambda[M]\right)=0 \tag{8}$$

The determinant is zero only at the set of discrete eigenvalues λ_i, or ω_i^2. The corresponding *i*-th eigenvector is $\{\phi\}_i$. So the eq. (3) can be rewritten as:

$$\left([K]-\omega_i^2[M]\right)\{\phi\}_i=0 \quad i=1,2,3\ldots \tag{9}$$

The number of eigenvalues and eigenvectors is equal to the number of degrees of freedom. In structural dynamics, each eigenvalue corresponds to the natural frequency and each eigenvector to the natural mode shape. Together they define one free vibration mode of the structure. The *i*-th eigenvalue λ_i is related to the *i*-th natural frequency as follows:

$$f_i=\frac{\omega_i}{2\pi} \tag{10}$$

where:

f_i - *i*-th natural frequency

$$\omega_i=\sqrt{\lambda_i}$$

The mode shape of a linear elastic structure that is in free or forced vibration, at any given time, is a linear combination of all of its natural mode shapes, or mathematically:

$$\{u\}=\sum_i\{\phi\}_i\xi_i \tag{11}$$

where:

$\{u\}$ - vector of physical displacements

$\{\phi\}_i$ - *i*-th natural mode shape

ξ_i - *i*-th modal displacement

There is a property of orthogonallity between eigenvectors. It is very easy to present the evidence for these properties. From eq. (9) with respect to the equation $\omega_i^2=\lambda_i$, for one vector $\{\phi\}_i$ is obtained:

$$[K]\{\phi\}_i=\lambda_i[M]\{\phi\}_i \tag{12}$$

and after pre-multiplying by another transposed vector, $\{\phi\}_j^T$ the previous equation becomes:

$$\{\phi\}_j^T[K]\{\phi\}_i=\lambda_i\{\phi\}_j^T[M]\{\phi\}_i \tag{13}$$

In the same way, starting from the vector $\{\phi\}_j$ and pre-multiplying by vector $\{\phi\}_i^T$ equation (12) becomes:

$$\{\phi\}_i^T[K]\{\phi\}_j=\lambda_j\{\phi\}_i^T[M]\{\phi\}_j \tag{14}$$

If $[K]$ and $[M]$ are real symmetric matrixes, it can be written:

$$\{\phi\}_i^T[M]\{\phi\}_j=\{\phi\}_j^T[M]\{\phi\}_i \quad \{\phi\}_i^T[K]\{\phi\}_j=\{\phi\}_j^T[K]\{\phi\}_i$$

Therefore, after subtracting eq. (14) from eq. (13) it is obtained:

$$0 = (\lambda_i - \lambda_j)\{\phi\}_i^T [M]\{\phi\}_j$$

And then if $(\lambda_i - \lambda_j) \neq 0$, i.e. if $i \neq j$:

$$\{\phi\}_i^T [M]\{\phi\}_j = 0 \text{ and } \{\phi\}_i^T [K]\{\phi\}_j = 0 \tag{15}$$

Eq. (15) define the orthogonal properties of the vectors with respect to the system mass or stiffness matrixes.

If $i = j$, then these two vectors are not orthogonal and then eq. (14) is equal to some non zero scalar constant:

$$\{\phi\}_j^T [M]\{\phi\}_j = m_j \,,\; m_j \text{ - } j\text{-th generalized mass} \tag{16}$$

and

$$\{\phi\}_j^T [K]\{\phi\}_j = k_j \,,$$

$$k_j = \omega^2 m_j \text{ - } j\text{-th generalized stiffness} \tag{17}$$

In addition, from eq. (16) and eq. (17), Rayleigh's equation is obtained:

$$\omega_j^2 = \frac{\{\phi\}_j^T [K]\{\phi\}_j}{\{\phi\}_j^T [M]\{\phi\}_j} \tag{18}$$

Eq. (15) are known as the orthogonal property of normal modes, which means that each normal mode shape is unique and different from any other. In addition, one normal mode shape cannot be obtained through a linear combination of any other normal mode shapes.

For practical examination natural mode shapes can be scaled. If one of the elements of the eigenvectors is assigned a certain value, the rest of the (n-1) elements are also fixed because the ratio between any two elements is constant. Thus, process of adjusting the elements of the natural mode shapes to make their amplitude unique is called normalization. The resulting scaled natural modes are called orthonormal modes. The mass (inertial) normalization is the default method of eigenvector normalization and scales each eigenvector, natural mode shape, to a unit value of the generalized mass:

$$\{\phi\}_i^T [M]\{\phi\}_j = 1 \tag{19}$$

When the structure is free, without support, then, besides elastic modes, it has rigid-body modes. The natural frequency of the rigid-body mode is equal to zero.

Methods for natural frequency and natural mode shape estimation

There are large numbers of numerical methods of the real eigenvalue extraction for its different type and size. While most of the methods can be applied to all problems, the choice is often based on the efficiency of the solution process.

The methods of eigenvalue extraction belong to one of the following groups [10]:

- Methods with direct egenvalue extraction finding zero determinants.
- Iterative methods (Inverse Iteration Method, Vector Iteration Method, Subspace Iteration Method) - In an iterative method, a certain number of roots are extracted at a time by iterative procedures applied to the original dynamic matrix.
- Transformation methods (Jacobi Method, Ritz Method, **Lanczos Method**, Givens Method, QL Hausholder Method, QZ and QR Method) - In the transformation method, the matrix of coefficients is first transformed into a special form (diagonal or tridiagonal) from which the eigenvalues may easily be extracted.
- Reduction method (Guyan Method) - In the reduction method, the structure is divided into substructures and after the eigenvalue extraction, they are synthesis again.

The characteristics of the Sturm series, the procedure for reortogonalization and deflection, can be used to improve methods. Some of these methods, as presented in Figure 2, are better for small degree of freedom systems (QL Hausholder Method, QZ and QR Method, Guyan Method and Jacobi Method), while the others (Subspace Iteration Method, Givens and Lanczos Method) are superior for large structures.

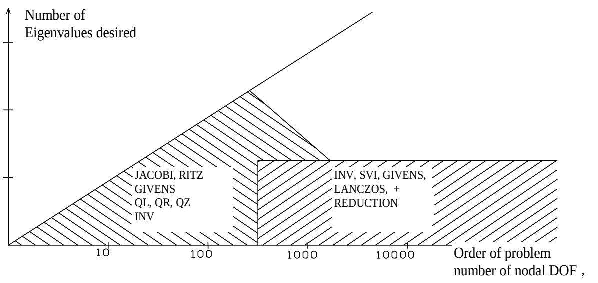

Figure 2. Selection of the method for eigensolution

When the number of degrees of freedom increases, then the necessary computer memory and time for extraction also increase. The Iterative Methods and the Lanczos Method are the most efficient for egenvalue extraction for the structures with a few hundred thousand degrees of freedom.

Lanczos Method

The Lanczos Method overcomes the limitations and combines the best features of the transformation and iterative methods [11]. It does not miss roots (transformation methods), and it only find the roots requested by the user (iterative methods). The Lanczos Method is the preferred method for most medium and large sized problems, since it has a performance advantage over other methods. In addition, the advantage is its capability of parallel processing.

In 1950, Lanczos published his procedure for efficient determination of m first or last in series egenvalues and appropriate eagenvectors of nxn symmetric matrix. However, not until 1975, Lanczos Method was included in software NASTRAN and in many other known FEM software [12]. Lanczos Method is included in the static and dynamic analysis software that is developed in theVTI.

The symmetric matrix [A] is obtained (eq. (4)) using certain transformation of the matrixes [M] and [K] so this procedure can be applied to natural frequency extraction. Pre-multiplying by the matrix $[V]^T$ and using the replacement $\{\phi\} = [V]\{y\}$, eq. (4) can be written as:

$$[V]^T[A][V]\{y\} = \lambda [V]^T[V]\{y\} \tag{20}$$

where,

$[V]$ - matrix of Lanczos vector v, size nxm

$\{y\}$ - new eigenvector dimension m

If it is assumed that Lanczos vectors [13] are orthonormal with respect to the mass matrix, $\{v\}_i^T\{v\}_j = \delta_{ij}$ (δ_{ij} - Kronecker delta), than $[V]^T[V] = [I]$ is valid, where $[I]$ is the unit matrix, eq. (20) can be transformed into the standard eigenvalue problem:

$$[T]\{y\} = \lambda\{y\} \tag{21}$$

$[T] = [V]^T[A][V]$, size mxm, and can be written as:

$$[T] = \begin{bmatrix} \alpha_1 & \beta_1 & & & & \\ \beta_1 & \alpha_2 & \beta_2 & & & \\ & \beta_2 & \alpha_3 & \beta_3 & & \\ & & \bullet & \bullet & \bullet & \\ & & & \beta_{m-2} & \alpha_{m-1} & \beta_{m-1} \\ & & & & \beta_{m-1} & \alpha_m \end{bmatrix} \tag{22}$$

The matrix $[T]$ is three diagonal and symmetric and the vectors v are defined in such a way to be orthogonal. When the matrix $[T]$ is defined as eq. (22), especially when it is mxm ($m<<n$) size, then eq. (21) can easily be solved using the direct determinant egenvalues calculation.

The Lanczos procedure for every new vector calculation uses the information that all previous calculated vectors contain. For the standard eigenvalue problem, a new vector is presented as the linear combination of j orto-vectors v, previously determined, and the residue $\{r\}_j$ that is orthogonal to all of them:

$$\{\bar{r}\}_j = [A]\{v\}_j\,,$$

or

$$\begin{aligned} \{\bar{r}\}_j &= \{r\}_j + \alpha_j\{v\}_j + \beta_{j-1}\{v\}_{j-1} + \\ &+ \beta_{j-2}\{v\}_{j-2} + \ldots + \beta_1\{v\}_1 \end{aligned} \tag{23}$$

Therefore, it is necessary to determine the coefficients $\alpha_j, \beta_{j-1}, \ldots, \beta_1$ and the vector $\{r\}_j$. After the Lanczos vector determination, eq. (23) can be rewritten as:

$$[A]\{v\}_j = \beta_j\{v\}_{j+1} + \alpha_j\{v\}_j + \beta_{j-1}\{v\}_{j-1} \tag{24}$$

or in a matrix form:

$$[A][V] = [V][T] + \beta_m v_{m+1}\{e\}^T \tag{25}$$

where, the vector $\{e\}^T = \{0 \quad 0 \quad 0 \quad \ldots \quad 1\}$ is size m. In the end, the matrix [T] is obtained from the equation $[V]^T[A][V] = [T]$.

LASTA Aircraft – Basic Information

The LASTA aircraft is intended for the basic figural, navigational, instrumental and night traning. It is shown in Fig.3.

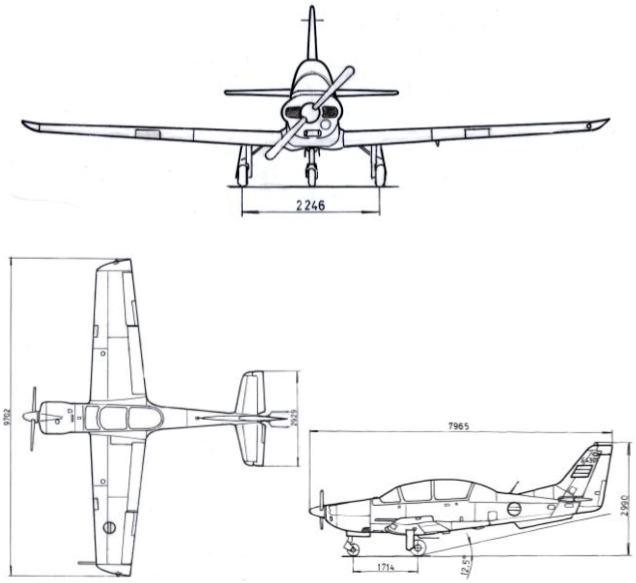

Figure 3. Aircraft LASTA

The LASTA aircraft is a low wing metal construction aircraft, with two, one behind another, classical non-ejection seats and a tricycle landing gear. The power group consists of one piston engine with six cylinders "AVCO-LYCOMING AEIO-540-L1.B5D" and two-blade propellers "HARTZELL HC-C2YR-4CF/FC 84756". The aircraft cabin area has heating and aeration. Windscreen defogging and de-icing is enabled. The LASTA aircraft uses concrete and prepared grass runways and posses possibility of independent launching.

Finite Element Model of the LASTA Aircraft

The first step in finite element modeling is to separate the structure, here aircraft, and view the rest as masses and forces. Second, the aircraft structure is divided to substructures - wings, fuselage, stabilizers, engines, landing gears, etc. Then each substructure is reduced into its components - rings, ribs, spars, cover plates, actuators, etc. It is continued through as many levels as it is necessary. Then, the element type and the element size are chosen before the beginning of the finite element modeling of the LASTA aircraft [14]. Elements have to be small enough to give more precise results, but, on the other hand, it is important that a total number of the degree of freedom would not be too much large because of the computer possibility. The size and type of elements are adapted to the structures. The aircraft structure is modeled by shell, beam and three-dimensional finite elements. All parts of the aircraft structure are modeled separately. The masses of the non supporting parts are modeled as the nonstructural or concentric masses. The structural parts are joined into subassemblies and then the stiffness calculation and the modal analysis are done. Finite element models of some substructures are shown from Fig.4 to Fig.7.

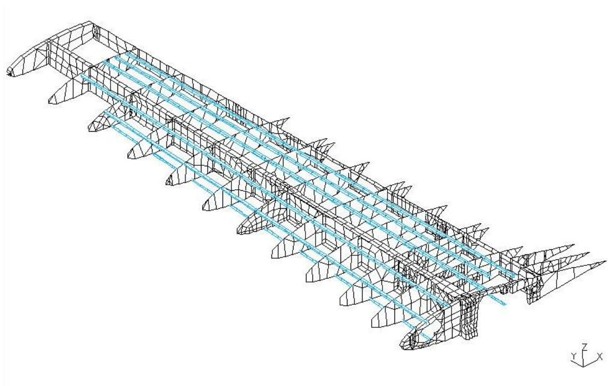

Figure 4. Wing rib and the stringer FE model

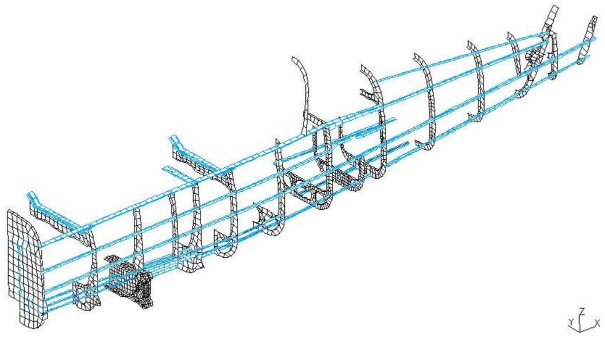

Figure 5. Fuselage frame and the stringer FE model

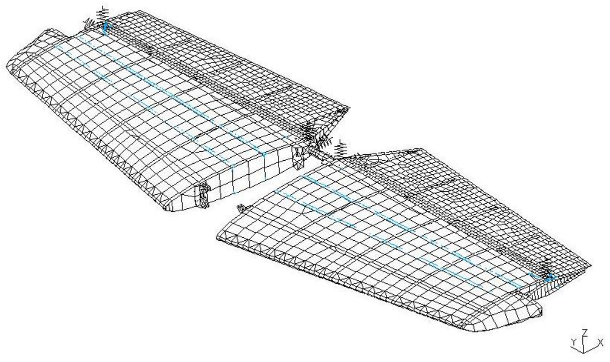

Figure 6. Horizontal stabilizer and the elevator FE model

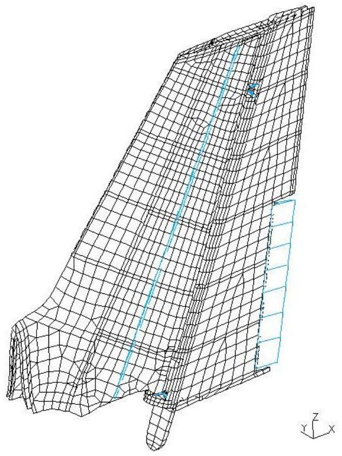

Figure 7. Fin and the rudder FE model

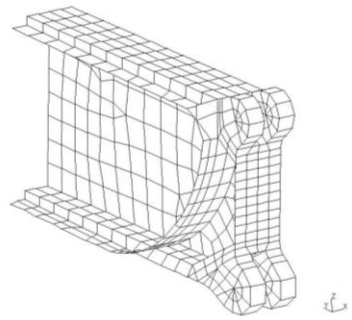

Figure 8. *Fitting FE model*

The main assembly of the aircraft finite element model is made by join subassemblies [15]. The subassemblies are connected in points where the tie-down fittings are positioned. These connections are modeled as beams and strings. In the end, the masses of pilots, fuel in reservoir, oil, equipment and flight controls are added to the aircraft FE model. The complete FE model of the LASTA aircraft has 42.129 finite elements, 37.367 nodes and 220.536 degrees of freedom and it is shown in Fig.8**.** The boundary conditions are free-free and the Shift Frequency Method is applied.

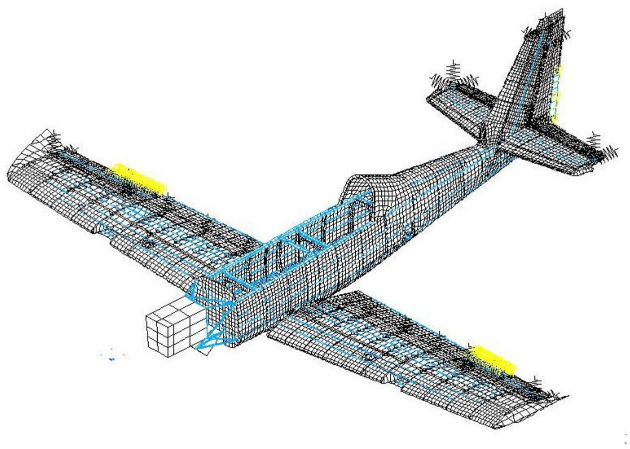

Figure 9. LASTA aircraft FE model

Computational Modal Analysis

The Computational Modal Analysis or the calculation of the natural frequencies and the natural mode shapes are done for the symmetric and anti-symmetric case according to the recommendation from [16]. The difference between these two cases is in the aileron string stiffness. The aileron string stiffness is k=200kN/m for the symmetric case and k=20kN/m for the anti-symmetric case.

The frequencies of the elevator and rudder rotation are fitted according to the command control circuits' stiffness obtained experimentally.

After the modal analysis, symmetrical natural mode shapes are chosen from the symmetric case calculation and anti-symmetric natural mode shapes from the anti-symmetric case calculation. All local mode shapes are neglected. All natural mode shapes are inertial normalized.

Natural Mode Shapes

The frequencies and mode shapes description of the finite element model of the LASTA aircraft are presented in Table 1. All natural mode shapes are shown from Fig.10 to

Fig.26.

Table 1. Natural mode shapes

No.	f (Hz)	Mode shape description	Figure
1	4.913	Aileron anti-symmetric rotation	Fig.9
2	7.316	Rudder rotation	Fig.10
3	8.140	Elevator rotation	Fig.11
4	11.397	First wing bending	Fig.12
5	11.745	Fuselage torsion	Fig.13
6	14.596	Aileron symmetric rotation	Fig.14
7	15.428	Horizontal stabilizer anti-symmetric bending	Fig.15
8	17.529	Fuselage symmetric bending in the vertical plane	Fig.16
9	20.688	Anti-symmetric coplanar wing and fuselage bending	Fig.17
10	22.711	Second wing bending	Fig.18
11	23.094	Symmetric coplanar wing	Fig.19
12	23.831	Horizontal stabilizer symmetric bending	Fig.20
13	24.367	Flap anti-symmetric rotation	Fig.21
14	24.759	Flap symmetric rotation	Fig.22
15	26.335	Fuselage anti-symmetric torsion	Fig.23
16	29.380	Fin bending	Fig.24
17	32.384	Wing symmetric torsion	Fig.25

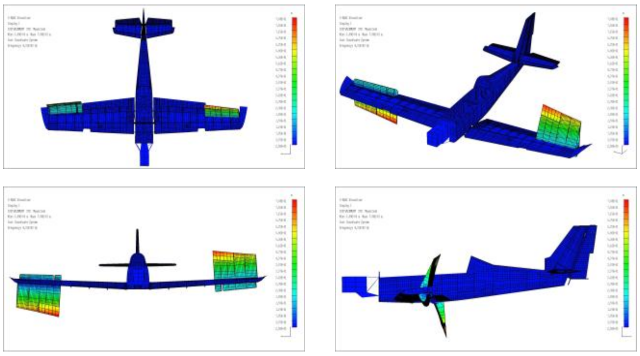

Figure 9. Mode 1 - Aileron aniy-symmetric rotation

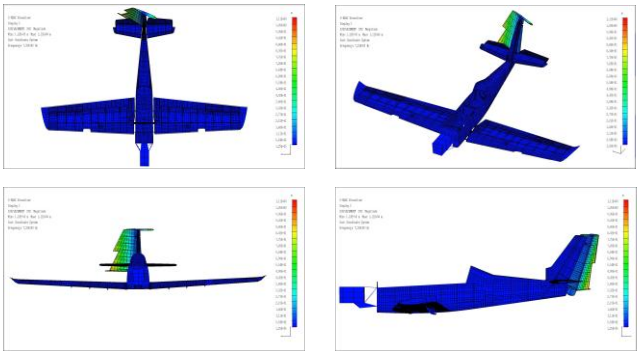

Figure 10. Mode 2 - Rudder rotation

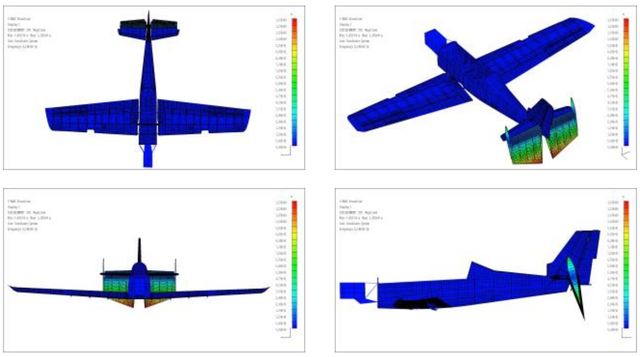

Figure 11. Mode 3 - Elevator rotation

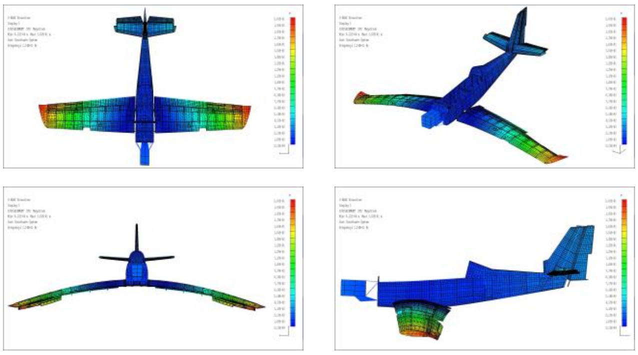

Figure 12. Mode 4 - First wing bending

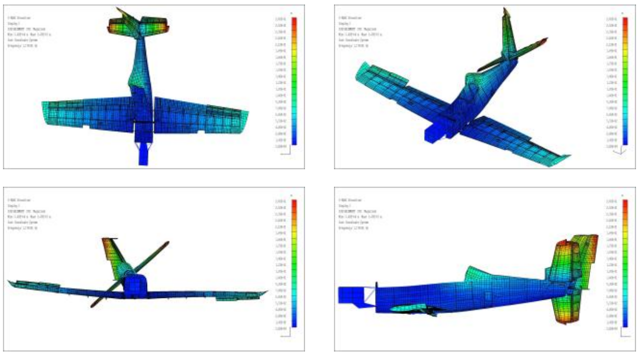

Figure 13. Mode 5 - Fuselage torsion

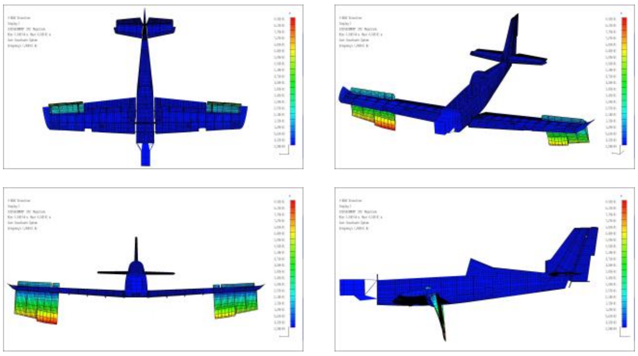

Figure 4. Mode 6 - Aileron symmetric rotation

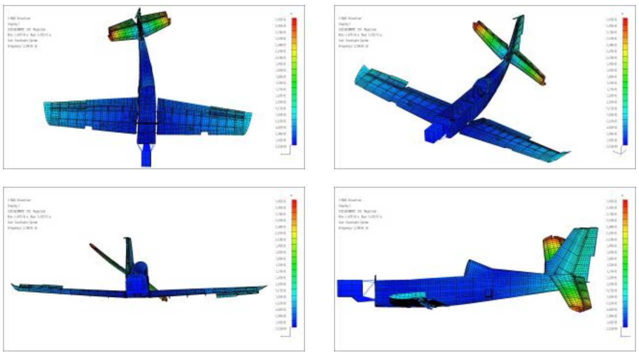

Figure 5. Mode 7 - Horizontal stabilizer aniy-symmetric bending

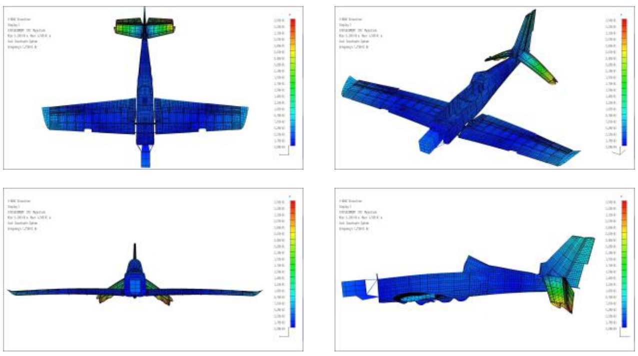

Figure 6. Mode 8 - Fuselage symmetric bending in the vertical plane

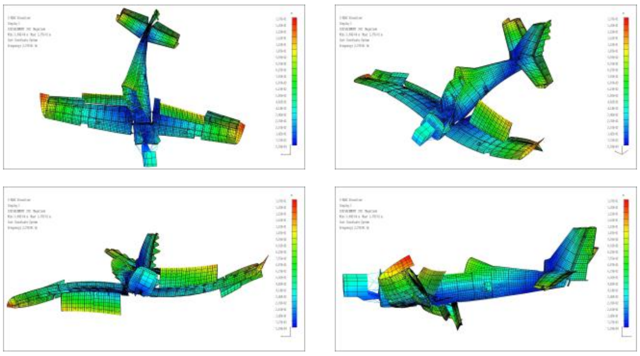

Figure 7. Mode 9 - Anty-symmetric coplanar wing and fuselage bending

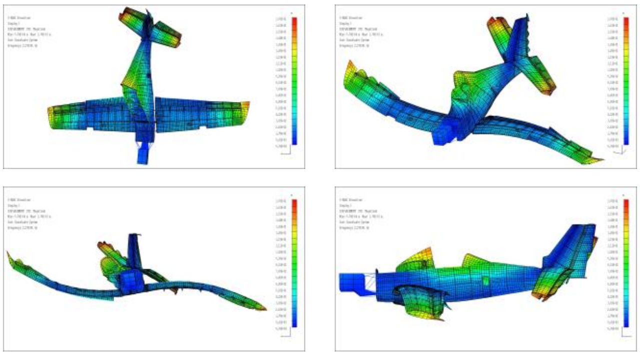

Figure 8. Mode 10 - Second wing bending

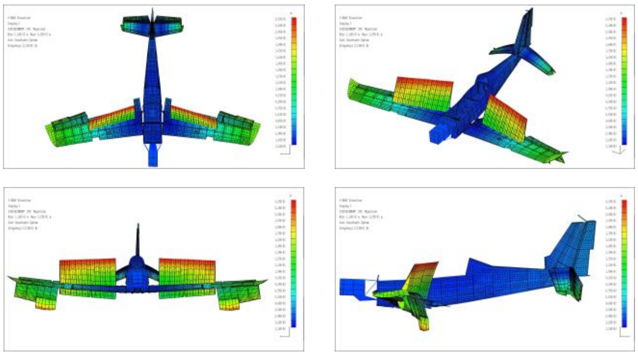

Figure 9. Mode 11 - Symmetric coplanar wing

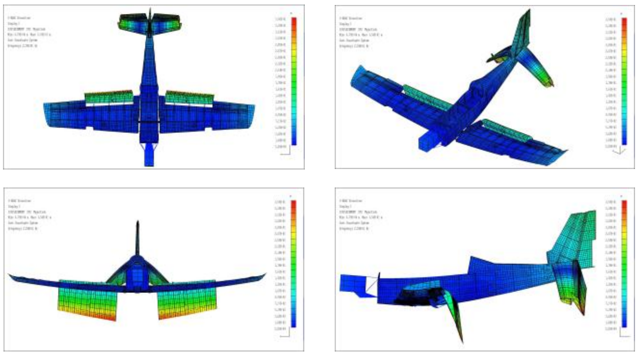

Figure 10. Mode 12 - Horizontal stabilizer symmetric bending

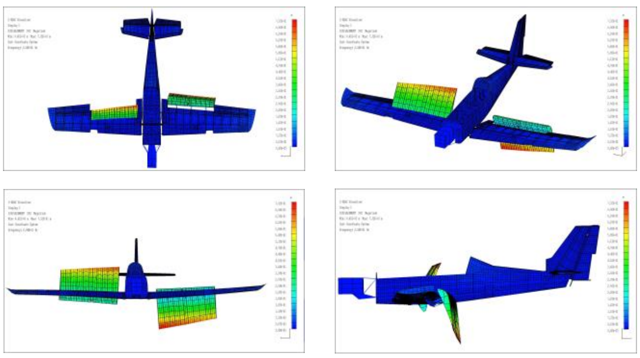

Figure 11. Mode 13 - Flap anti-symmetric rotation

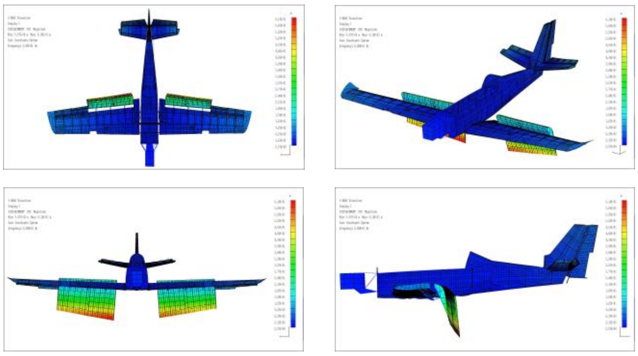

Figure 12. Mode 14 - Flap symmetric rotation

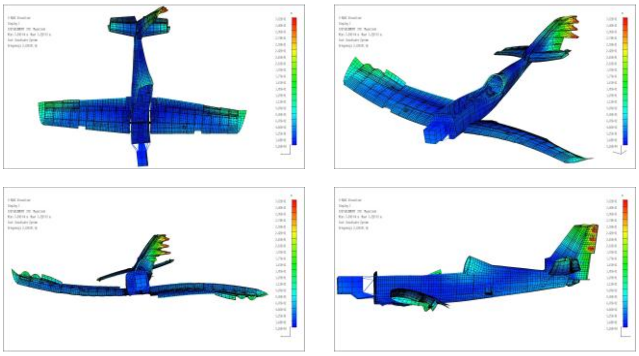

Figure 13. Mode 15 - Fuselage aniy-symmetric torsion

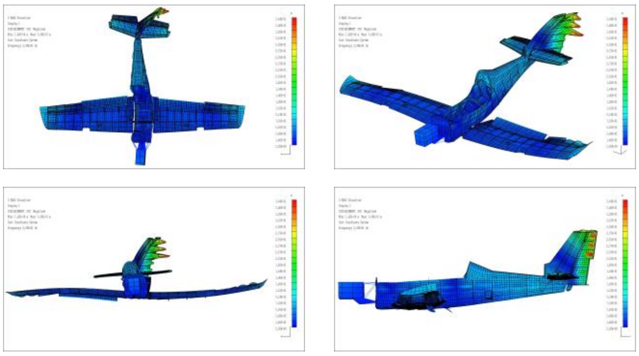

Figure 14. Mode 16 - Fin bending

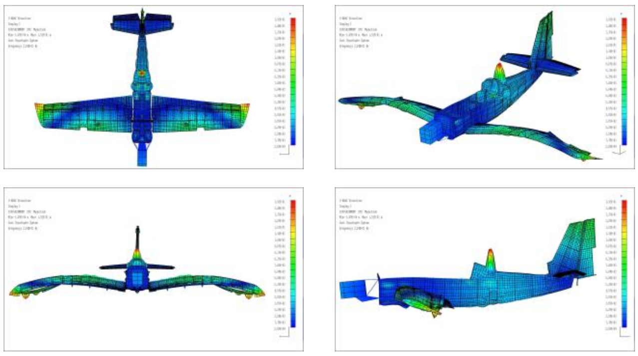

Figure 15. Mode 17 - Wing symmetric torsion

Conclusion

The results of the Computational Modal Analysis of the LASTA aircraft are presented in this paper.

All aircraft parts are modeled by finite elements and they are joined into the aircraft finite element model that has 220.536 degrees of freedom. The natural frequencies and the natural mode shapes of seventeen modes are obtained by the Computational Modal Analysis and they are presented in Table 1. All natural mode shapes are shown from Fig.9 to Fig.16.

The results of the Computational Modal Analysis for the LASTA aircraft will be used for the calculation of unsteady generalized aerodynamic forces and critical flatter speeds. The natural frequencies and the natural mode shapes demonstrate where to expect modes on the Ground Vibration Test (GVT) – Experimental Modal Analysis (EMA).

A real value of the natural frequencies, the natural mode shapes and the damping will be obtained after the Ground Vibration Test. These results will be then used for the final calculation of the unsteady generalized aerodynamic forces and critical flatter speeds. In addition, according to these results of frequencies and damping, the aircraft finite element model will be verified and fit for further use in the analysis of the aeroelastic phenomena (aeroelastic twisting, reverse, critical flutter speed calculation, aeroelastic response to gust, etc.) and aircraft dynamic response (under armament action, landing, ground roll and taxiing).

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Računska modalna analiza aviona LASTA

U ovom radu su prikazani rezultati računske modalne analize kompletnog modela konačnih elemenata aviona LASTA. Rezultati proračuna, sopstvene frekvencije i sopstveni oblici oscilovanja, koji su ovde prikazani, biće korišćeni za dalju analizu aeroelastičnih pojava radi provere njegove bezbednosti od pojave flatera u okviru anvelope upotrebnih brzina leta.

Ključne reči: **avion, elisni avion, školski avion, sopstvene oscilacije, modalna analiza, numerička analiza, metoda konačnih elemenata.**